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# The parable of the broken pencil or syntactic incidents and their consequences

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*When using technology, the translation of a mathematical concept into a particular technology brings syntactic difficulties that may lead to problems that can either be an excuse to withdraw from knowledge construction or a starting point for a mathematical reflexion. The boundary between the two attitudes is directly linked to the situation and to the integration of technology within the classroom. In this paper, I'll present the notion of syntactic incident and show in a particular class situations how students react in front of the consequences of such an incident.*

**Keywords**: Technology, didactical incidents, formative assessment, theory of didactic situations.

## INTRODUCTION

When Evelyne decided to begin her novel after a long reflexion about the characters she wanted to introduce, the context in which they will evolve and the general scenario of the story she wanted to share with the world, she sat down in front of her desk, laid down a sheet of white paper, verified that her eraser was on the desk and took a pencil. "Once upon a ti..." she wrote. And her pencil broke. She first searched for a pencil sharpener but she didn't find one (she remembered having given it to her nephew for the beginning of the school year); after a thundering cry of anger, she decided:

1) to write with a pen instead of a pencil even if it will not be possible to erase her writing,

2) to take a knife and to try to sharpen her pencil even if the lead will not be sharp enough for her writing,

3) to stop writing of the first page of her novel and to go shopping (buy a pencil sharpener, she thinks)

4) to sit down in her chair thinking about the incipit of her novel and remembering the most famous incipits of the literature... "Longtemps, je me suis couché de bonne heure...", "Call me Ishmael.", "The sun shone, having no alternative, on the nothing new", "I have never begun a novel with more misgiving"... [1]

5) to call a friend asking for a pencil sharpener,

6) to abandon this adventure which is not for her, even the things are against her,

7) to go and buy a computer, it will surely be safer to write her novel and she has always thought that it is high time to understand computer science,

8) ...

An incident occurred thwarting her intentions and this incident is directly linked with the tool she wanted to use with precise intentions: she wrote with a pencil because she wanted to erase her clumsiness and to keep a clean manuscript. The next episode of this story may be very different regarding the different attitude she would adopt.

In the first case, a direct consequence will be that she will not be able to erase the first draft. If she wants to keep the idea of a clean manuscript, she'll have to think more carefully to the sentences she'll write, otherwise she'll have to strike through her draft but, in doing so, she'll keep in memory her different trials. The choice of the tool she'll use has consequences on the organization of the content. In the same time she loose properties of the first tool, she gains new ways of writing.

The second attitude will modify the potentialities of her tool: she chose a pencil instead of a pen or a quill

for some reasons, rational or irrational. But she surely had built for a while schemes of utilization of this particular tool and the combination of an unprecedented sharpness of detail and smoothly flowing movement will disappear.

In the third case, there is a break in the continuity of her work. She perhaps will come back to this particular state of mind that allows to begin the writing of a story, but for the moment, the incident stopped her progression. This important disturbance modifies deeply the next step of the story: instead of writing, she goes shopping and perhaps, she may come back to her task later.

The fourth case is also a break in the progression of her writing, but in that case, the new direction she takes, brings her in a deep reflexion about her own writing in the literature's world. She stays in the general context of her task but still work on the first sentence in another way she did initially.

In the fifth case, there is an externalisation of the procedure. The solitude of the writer is broken by an external component. She will have to explain and perhaps to justify her will of writing a novel, she'll surely have to summarize her first ideas, to introduce the theme of her novel... The first environment that she designed is deeply modified.

This sixth case is surely the most radical case where the incidents lean to an abandonment of the realization of the task. It is certainly because the project was not deep enough within the writer's mind, and using didactical words, that the devolution of the task is not made.

The seventh case is also a very radical transformation of the writing conditions. A result of the incident is to consider that the tool is not appropriated for the task and that the learning of a new way of writing is a precondition to complete the project successfully.

There are surely other developments of this story, but the lesson of this parable is that, regarding the conditions, the actors, the environment, a same event can lead to different scenarii that may modify the continuity of a story. Taking into account this parable in the field of mathematics learning can give interesting tools to analyse the mathematical activity of students in front of a specific task.

More theoretically, when students are learning in a digital environment that is the result of a construction of the teacher who has specific intentions and of students' knowledge regarding both technology and mathematics, the questions relative to the syntactic knowledge and interpretation are crucial for the construction of mathematical knowledge at stake.

More precisely, a question that can be addressed and that will be developed in the paper concerns the conditions that allow students to overcome syntactic incidents in order to transform them into mathematical questions and lean to knowledge construction.

## THEORETICAL CONSIDERATIONS

In order to answer such questions, the first approach is to precise in which context the answers will be searched. In our case, the methodology is based on the theory of didactic incidents (Aldon, 2011) which took its founding principles in the theories of didactic situations (Brousseau, 2004) and of instrumental genesis (Rabardel, 1995; Artigue, 1997; Trouche, 2004; Drijvers & Trouche, 2008).

The theory of didactic situations takes, as a starting point, the relationship between an interaction of a player in a particular game with his/her *milieu*, and knowledge. The didactic situation is for an observer the modeling of the environment of the game, and is the game itself for the student. From the point of view of students, the environment of the game is integrated within the game and knowledge construction results from the interaction of the player with the entire environment including his/her own knowledge, mathematical situation given through a specific wording, the interactions with the teacher and available artefacts at this moment. Interactions of the player with the environment produce knowledge through the experiences build with the different parts of this environment. An important point is to consider in this environment the different available artefacts and the process of transformation of these artefacts into instruments useful for winning the game. The instrumental genesis theory that initially comes from ergonomic studies considers the artefact as a thing without any intentions. The use of the artefact in specific context transforms it slowly into an instrument that can be considered as the combination of an artefact and schemes of utilization. The integration of technology into the classroom is of the same nature and

can be considered as a slow process in which the given technology (the artefact) becomes an instrument through the double movement of instrumentation and instrumentalization. The instrumentation is the process where the artefact modifies the subject's activity and the instrumentalization is the process where the subject modifies the artefact for her own use.

A didactic incident is defined as "an event of the didactical system that occurs sporadically, that is unforeseen, and that requires an appropriate answer of the actors" [2] (Aldon, 2011, p. 26). A didactical system is the implementation of a didactic situation in a particular context. In the different didactic incidents that have been picked out, some of them are directly linked to the digital environment of the didactic situation. A syntactic incident is a problem that occurs in the conversion from a register of representation in another. The term *syntax* refers to the units that make up rules to accomplish an action. For example, the drawing of a line in the "language" of paper and pencil can be done using a ruler and a pencil (place the ruler on the paper, place the pencil along the ruler and follow the ruler with the pencil) and, this same drawing in the language of a GDS will be to choose the menu create a point, to show with the mouse the place of the point, to click with the right button of the mouse, to choose the menu line, etc.

Especially, when there is a translation into a digital representation, syntactic incidents can *a priori* be triggered by two factors: the operation is not foreseen by the software and must be built or the operation is provided but doesn't work due to a misunderstanding of the syntax of the command or, is not known by the operator, in a particular environment. The perturbations that follow can be short in the case of an assistance provided by an actor of the situation (teacher, other student, element of the *milieu*...) or by understanding of the phenomenon by the subject (which is part of his/her instrumentation). But, they may also have long-term consequences, as shown in the parable of the writer who breaks her pencil: a disengagement of the subject locally (3) or globally (6) leading to a new instrumental genesis, a loss of devolution of the situation (3, 6, 7) that may lead to a disengagement of the student, a questioning of the relevance of the artefact in achieving the task (1, 2, 7) that may come out to a reflection about the pertinence of a tool relatively to a mathematical task or a mathematical concept, a modification of the working environment (5) and a reorganization of the way to approach the problem (4) whose consequence could be either the beginning of a new learning different from the teacher's intention or a new approach allowing knowledge construction.

We took the opportunity of a wide introduction of handheld technology in different classes to study the impact of such a technology on teaching and learning of mathematics. The example that is developed in the next paragraph comes from a class which is equipped with TI Nspire calculators. The teacher (J.L. in the following) worked in this class in a perspective of inquiry-based learning and proposed to his students (16 years old, in a scientific major) problems that allow a personal and collective reflection. The example of this paper has been designed, observed and discussed in the context of the European project EdUmatics [3] and illustrate a syntactic incident leading to different perturbations.

## EXPERIMENTATIONS

The context of the experimentation is a mathematical problem that is proposed to the students. The general context of this problem is the study of the relationship between the distance of a walker to a given point according to his/her position on a path. In this study, we follow students who are searching the following problem:

Pjotr moves at constant speed along a square $ABCD$ with center $O$ (intersection of the two diagonals) and given side $L$, starting from vertex point $A$.

Pjotr wants to describe how his distance from the center $O$ of the square changes while he is moving along the square. How can you help him/her? (If you wish, you can choose any positive real number as the length $L$ of the side of the square).

The observed group worked either on their own calculators or on a computer with the software TI Nspire. The different illustrations of this paper show the camera viewpoint for this group of four students who are working both with their own calculator and with the software available on computer. The first analyzed episode comes after a first episode of work with paper and pencil that leads students to explicitly calculate the relationship of the distance that Pjotr covered along the first side of the square and his distance to the center as shown on the Figure 1.

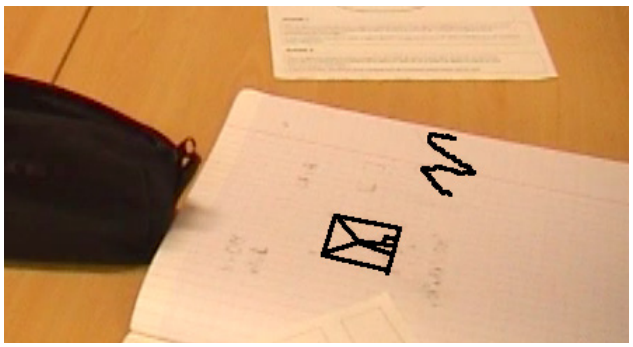

**Figure 1:** The first approach of the problem with paper and pencil

It is interesting to notice, that a freehand drawing of the behaviour of the function (which has been stressed on the Figure 1) has been made on the paper before any calculation: "*it'll be like that, it'll make a wave*" a student said drawing the freehand drawing. The second episode of work consists in a transposition of the mathematical situation to the software. The idea developed by the four students was to capture in the spreadsheet the values of the distance of *P* (the position of Pjotr on the square) to the centre of the square and to represent on a graphic these values according to the covered distance of point *P* from his starting point *A*. The syntactic incident comes from the translation of this idea in the language of the software. It is indeed possible to measure the distance of two points but there is no menu giving the distance from a point to another on a given path (here the square *ABCD*). The first trial was indeed to consider the abscissa of the graphic representation as the distance of *A* to *P*. The resulting graphics appeared to be in contradiction with the idea expressed previously and drawn on the paper. In that case, the syntactic incident associated with the previous mathematical reflexion led one of the students (S1) to think differently the parametrization of the point *P* in this task. Students are here in the fourth situation of the parable: the incident is the starting point of a new mathematical reflexion that comes to a definition of a piecewise function:

J.L.: And how did you do to obtain the second arch?

S1: In the next drawing, I made BP but each time I added six because six is the length AB...

In the same time and in front of the same situation, the second student (S2) working on his handheld tried to solve the same problem. However, his mastery of the technology or the mathematical reflexion are not sufficient to allow a transformation of his research strategy. In the contrary, most of his time was spent to try to solve the syntactic problem using the function "distance of two points" and without referring to the mathematical situation. The result was in contradiction with the former reflexion but this student tried to solve the problem without the good tool. He was exactly in the situation of the second choice of the parable: the tool is not adapted to a precise goal but can give the illusion to work well. The result of the work is illustrated on the handheld screen of the Figure 2. In the didactic situation, there was, in that case, what Margolinas (2004) called a didactic bifurcation where the problem that try to solve the student is no more the problem that the teacher wanted to be solved. It is also an illustration of the seventh choice of the parable: there is a knowledge construction or a trial of knowledge construction but out of the intentions of the teacher. Even if this knowledge participates to the instrumental genesis of the student, the lack of institutionalization leads the student to consider this trial as a failure. It's not sure that the different trials and errors done by this student lead to a better understanding of the technology and, in the contrary it could be a pretext to abandon this technology which is "*a waste of time*" as expressed by another student in an interview:

Interviewer: [...] and do you remember the time you said, oh no, I do not want any more this calculator?

Student: It was very early, yes because we had to go to the menu, go to this location there, finally, Click everywhere, we had quite a

**Figure 2:** Two different consequences of the same syntactic incident

> journey to make a calculation you could do very easily with our old calculator, in fact faster. [4]

In this paper, we only study one example of syntactic incident in a particular episode, but, it is by studying different observations that we have been able to determine the types of behaviour described in the parable of the introduction.

## MAIN FINDINGS AND CONCLUSION

One syntactic incident and its consequences on two different students has been detailed in the previous section showing that the perturbations following an incident may differ regarding the choice that follows the incident. The question that we posed in the introduction was to analyze the reasons why one or another consequences occur and more precisely why and when an incident is a starting point of a reorganization of knowledge or not. Answers to these questions are interesting for the student as well as for the teacher in a perspective of formative assessment, defined by Bell and Cowie (2001, p. 536) as "the process used by teachers and students to recognize and respond to student learning in order to enhance that learning, during the learning". The European FP7 project FaSMEd [5] "aims to research the use of technology in formative assessment classroom practices in ways that allow teachers to respond to the emerging needs of low achieving learners in mathematics and science so that they are better motivated in their learning of these important subjects." The incident analysis is part of the toolkit that allows to understand better the behavior of students in a digital learning environment.

In the different observations in teaching and learning digital environment, the analysis of syntactic incidents and the perturbations that follow show different cause that can be clues for students and for teachers in order to understand when and why knowledge at stake in a particular mathematical situation is not reached.

Incidents lead to perturbations that prevent knowledge construction when:

- Technology is external to mathematics, that is to say technology is not included in the set of mathematics tools useful in the resolution of a mathematical problem for a given student at a certain moment.
- The knowledge of the syntax overcomes mathematical knowledge: in that case learning the syntax (in the sense defined in the second section) adds technical or conceptual difficulties that lead students to forget the mathematical notions at stake.
- The technological knowledge is more difficult that math knowledge at stake in a given situation: in that case, it is important to think about the adapted technology.
- Technology that is used doesn't supplies potentialities that are necessary in the mathematical situation most of the time because of a bad initial choice of technology.

In the contrary, incidents lead to construction of knowledge when:

- The class culture takes into account the experimental part of mathematics, and technology is used internally in different mathematical situations.
- The instrumental genesis is sufficient to give adapted technological skills in the learning situation.
- The knowledge of the syntax of a particular technology is either sufficiently natural or trained before having to use it in a complex mathematical situation

A continuation of this study will be to make this kind of analysis operational both for teachers in a perspective of formative assessment and for students in a perspective of auto-evaluation. This work is part of the FaSMEd project.

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## ENDNOTES

1. Proust, *A la recherche du temps perdu*, Melville, *Moby-dick*, Beckett, *Murphy*, Somerset Maugham, *The razor's edge*.

2. "un événement du système didactique qui se produit de manière irrégulière, non prévu, nécessitant des acteurs une réponse appropriée" (Translated by us).

3. 50324-UK-2009-COMENIUS-CMP; European Development for the Use of Mathematics Technology in Classrooms, http://www.edumatics.eu

4. I: et c'est à quel moment, vous vous souvenez le moment où vous avez dit, ah non, je ne veux pas de cette calculatrice ?

E: C'était très rapidement, oui parce qu'il fallait aller dans le menu, aller dans cet endroit-là, enfin cliquer de partout, on avait pas mal de cheminement pour faire un calcul qu'on pouvait très bien faire avec notre calculette, plus rapidement en fait. (Aldon, 2011, p. 652) (Translated by us).

5. Grant agreement no: 612337, Improving progress for lower achievers through Formative Assessment in Science and Mathematics Education. http://research.ncl.ac.uk/fasmed/